Acoustics off buildings

In day today life sound engineering plays a vital role in film industries, broadcasting of television signals and even in television signals. So a new field of science is developedwhich deals with the planning of a building or a hall with a view to provide best audiblesound to the audience and is called Acoustics of building.

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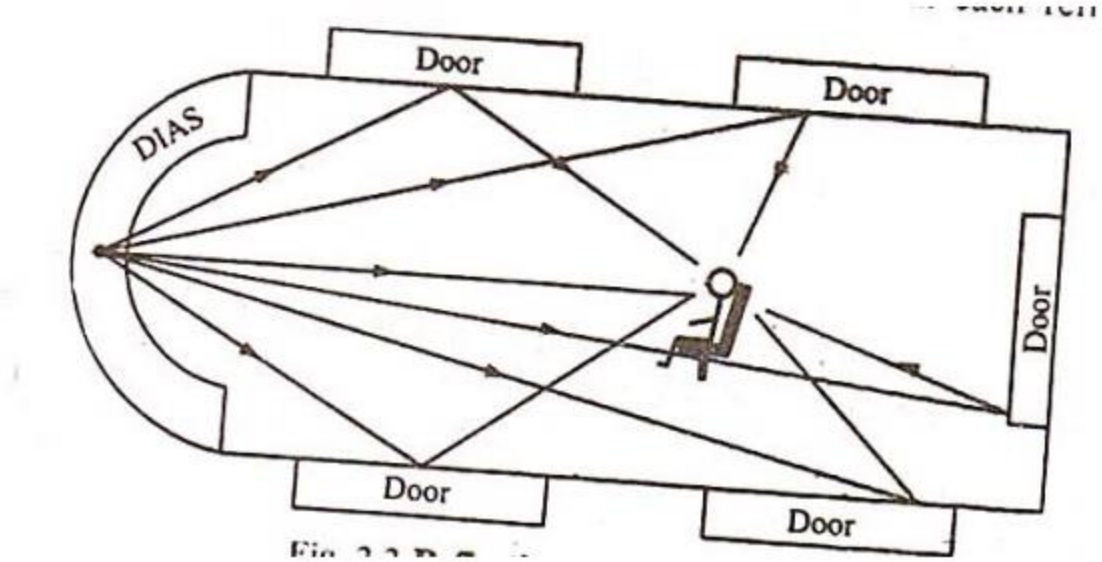
In day today life sound engineering plays a vital role in film industries, broadcasting of television signals and even in television signals. So a new field of science is developedwhich deals with the planning of a building or a hall with a view to provide best audiblesound to the audience and is called Acoustics of building. Therefore to provide a bestaudible sound in a building or hall a prime factor called Reverberation.

1 REVREBERATION



When a sound pulse is generated in a hall, the sound wave travels towards all directionand are reflected back by the walls, floors, doors, windows ceiling etc as shown in the figure.

So a sound wave has two to three hundred repeated reflections, before it becomes inaudible. Therefore, the observer in the hall does not be able to hear a singlesharp sound instead a "role of sound" of diminishing intensity (since part of energy is lostat each reflection)

2 Reverberation time

The duration for which the sound persist is termed as reverberation time and is measured as the time interval between the sound produced by the source produced by the source and to the sound wave until it dies.

Definition:

It is defined as the time taken for the sound to fall below the minimum audibility measured from the instant when the source sound gets stopped.

In designing the auditorium, theatre, conference halls etc, the reverberation time is the key factor.

If the reverberation time is too large, echoes are produced and if the reverberation time is too short it becomes inaudible by the observer and the sound is said to be dead.Therefore the reverberation time should not be too large or too short rather it should havean optimum value.

In order to fix this optimum value standard forumla is dervied by W.C.Sabine, who defined the standard reverberation time as the time taken for the sound to fall to one millionth of its original intensity just before the source is cut off.

$E = \frac{E_m}{10^6}$ Where E is the energy or intensity of sound at any time't'

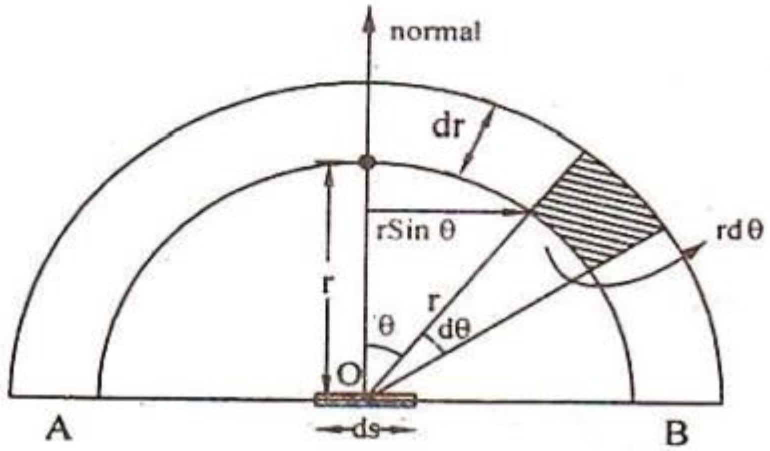
E_m = the maximum sound energy produced originally

3 SABINES FORMULA FOR REVERBERRATION

The relation connecting the reverberation time with the volume of the hall, the area and the absorption coefficient is known a s Sabine's Formula.

Sabine's developed the formula to express the rise and fall of sound intensity by the following assumptions.

I. Distribution of sound energy is uniform throughout the hall
II. There is Interference between the sound waves.
III. The Absorption coefficient is independent of sound intensity.
IV. The Rate of emission of sound energy from the source is constant.



Let us consider a small element ‘ds’ on a plane wall AB. Assume that the element ds receive the sound energy ‘E’.
Let us draw two concentric circles of radii 'r' and r + dr from the center point 'O' of
ds. Consider a small shaded portion lying in between the two semi circles drawn at an angle θ and θ+dθ, with the normal to ds as shown in the figure.

Let 'dr' be th radial length and rdθ be the arc length

Area of shaded portion rdθdr ---- (1)

If the whole figure is rotated about the normal through an angle ‘dϕ’ as shown in the figure, then it is evident that the area of the shaped portion travels through a small distance dx.

$$dx = r\sin\theta.d\phi \longrightarrow 2$$

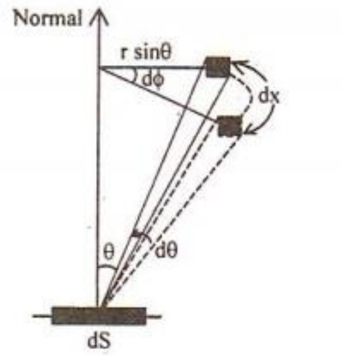
Therefore Volume of the shaded portion is

dV = Area$\times$ distance

Substituting from equation 1 and 2 we have

$$dV = rd\theta dr.r\sin\theta d\phi$$

$$dV = r^2\sin\theta dr.d\theta d\phi$$

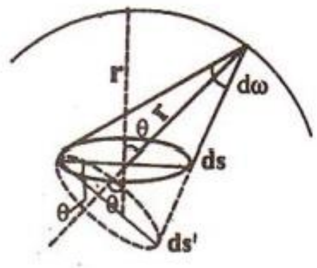


$\therefore$ The sound energy present in this volume

$$dV = E\times volume$$

$$dV = Er^2\sin\theta dr.d\theta d\phi$$

This sound energy will travel through the element in all directions.



$\therefore$ The sound energy present in this volume dV per unit solid angle is

$$dV = \frac{Er^2\sin\theta dr.d\theta d\phi}{4\pi}$$

$\therefore$ In this case the solid angle subtended by the area 'ds' at this element of volume 'dV' is

$$d\omega = \frac{ds'}{r^2}$$

From figure we can write

$$\cos\vartheta = \frac{ds'}{ds}$$

$$ds' = ds\cos\theta$$

Therefore we can write solid angle subtended by the area 'ds' as

$$d\omega = \frac{ds'}{r^2} = \frac{ds\cos\theta}{r^2}$$

Hence, the sound energy travelling from the element (i.e., from dw to 'ds')

$$= \frac{Er^2\sin\theta dr.d\theta d\phi}{4\pi}\frac{ds\cos\theta}{r^2} \longrightarrow 3$$

To find total energy received by the element 'ds' per second, we have to integrate the equation 3 for the whole volume lying within a distance 'v' is the Velocity of sound.

It is obvious from the geometry of the figure that,

It is obvious from the geometry of the figure that,

ϕ Changes from 0 to 2π

θ Changes from 0 to $\pi/2$

r Changes from 0 to v

$\therefore$ Integrating equation 3 with respect to these lines, we can write

Energy received per second by ds = $\dfrac{Eds}{4\pi}\int_0^v dr \int_0^{\frac{\pi}{2}} \sin\theta.\cos\theta.d\theta \int_0^{2\pi} d\phi$

$$=\frac{Eds}{4\pi}2\pi\int_0^v dr\int_0^{\frac{\pi}{2}}\sin\theta.\cos\theta.d\theta$$

$$=\frac{Eds}{2}\int_0^v dr\int_0^{\frac{\pi}{2}}\sin\theta.\cos\theta.d\theta$$

Multiply both numerator and denominator by 2 we get

$$=\frac{Eds}{4}\int_0^v dr\int_0^{\frac{\pi}{2}}2\sin\theta.\cos\theta.d\theta$$

$$= \frac{Eds}{4} \int_0^v dr \int_0^{\frac{\pi}{2}} Sin2\theta . d\theta$$

$$= \frac{Eds}{4} \int_0^v dr \qquad \because \int_0^{\pi/2} Sin2\theta = 1$$

$$= \frac{Eds}{4} v \longrightarrow \qquad 4$$

Let 'a' be the absorption coefficient of the area 'ds'.

$\therefore$ The energy absorbed by ds in unit time = $\frac{Evads}{4}$

$\therefore$ The total absorption at all the surfaces of the wall is $\frac{1}{4} Ev \sum a.ds$

$\therefore$ Total rate of Energy absorption = $\frac{1}{4} EvA \longrightarrow$ 5

Where 'E' is the energy from sources and 'A' is the total absorption on all surfaces on which the sound falls $A = \sum a.ds$

3.1 Growth and Decay of Sound Energy

If 'P' is the Power Output (i.e., the rate of emission of sound energy from the source) then we can write

Rate of Emission of Sound energy i.e., Power Output $P = \frac{1}{4} E_m vA$

Here E_m is the maximum energy from the source (which has been emitted) that is maximum energy which is incident on the wall

$$\therefore E_m = \frac{4P}{vA} \longrightarrow \quad 6$$

If V is the volume of the hall we can write the total energy at any instant't' = EV

$\therefore$ Rate of Growth or Increase in energy $= \frac{d}{dt}(EV) = V\frac{dE}{dt} \longrightarrow \quad 7$

At any instant

Rate of Growth of Energy	=	Rate of Supply of Energy from the source	-	Rate of absorption of energy by walls

$\therefore$ From equation 5 and 7 we can write

$$V.\frac{dE}{dt} = P - \frac{1}{4}EvA$$

$$\frac{dE}{dt} = \frac{P}{V} - \frac{EvA}{4V}$$

Let $\alpha = \frac{vA}{4V}$

$$\frac{dE}{dt} = \frac{4P}{vA}\alpha - \alpha E$$

$$\frac{dE}{dt} + \alpha E = \frac{4P}{vA}\alpha$$

Multiplying by $e^{\alpha t}$ on both sides, we have

$$\left(\frac{dE}{dt} + \alpha E\right)e^{\alpha t} = \left(\frac{4P}{vA}\alpha\right)e^{\alpha t}$$

$$\frac{d}{dt}\left(Ee^{\alpha t}\right) = \frac{4P}{vA}\alpha e^{\alpha t}$$

Integrating an solving we get

$$Ee^{\alpha t} = \frac{4P}{vA}.e^{\alpha t} + k \longrightarrow \quad 8$$

Where k is the constant of integration

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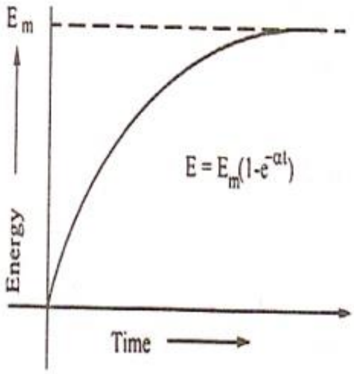
3.2 Growth of Sound Energy

Let us evaluate for growth
Initially during the growth the boundary conditions
Are at t=0 E=0
Thereore equation 8 becomes



$$0 = \frac{4P}{vA}.1 + k$$

$$k = -\frac{4P}{vA}.$$

Therefore substituting the value of k in equation 8 we can write

$$Ee^{\alpha t} = \frac{4P}{vA}.\alpha e^{\alpha t} - \frac{4P}{vA}$$

$$Ee^{\alpha t} = \frac{4P}{vA}\left(e^{\alpha t} - 1\right)$$

Since from equation 6 $E_m = \frac{4P}{vA}$, we can write

$$Ee^{\alpha t} = E_m\left(e^{\alpha t} - 1\right)$$

Dividing by $e^{\alpha t}$ throughout, we get

$$E = E_m\left(1 - \frac{1}{e^{\alpha t}}\right) \longrightarrow 9$$

Where E_m is the maximum sound energy.
This expression gives the growth of sound energy density ‘E’ with time ‘t’. The growth is along an exponential curve as shown in the figure.

This indicates that E increases until $t = \infty$

At $t = \infty$; $E = E_{max}$

3.3 DECAY OF SOUND ENERGY

Let us irst evaluate k or decay.
Here the boundary conditions are at t=0; E=Em

Initially the sound increases from E to E_m and now it is going to decay from Em. Therefore time is considered as '0' for E=Em. At $E=E_{mv}$ the sound energy from the source is cut off. Therefore rate of emission of sound energy from the source=0 i.e., P=0

Therefore from equation 8 we can write

$$E_m e^0 = 0 + k$$

$$k = E_m$$

Therefore substituting the value of k for decay in equation 8 , we get

$$Ee^{\alpha t} = \frac{4P}{vA} e^{\alpha t} + E_m$$

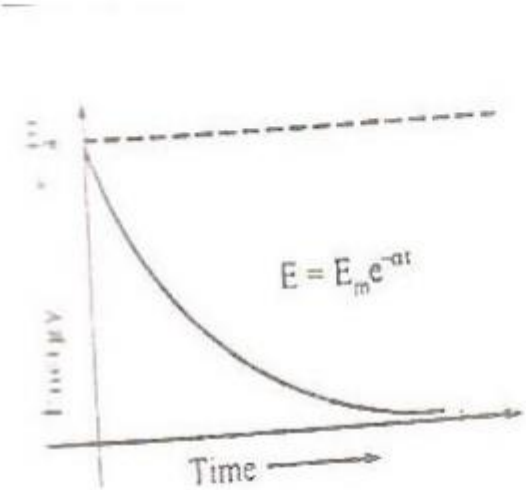
Since P = 0 Energy from the source is cutoff for decay of sound.

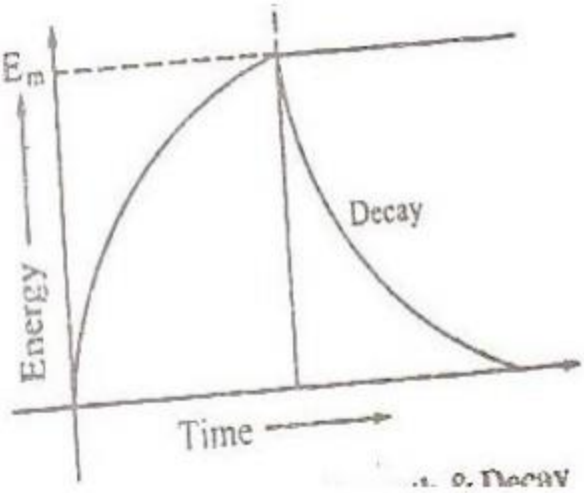
We can write

$$Ee^{\alpha t} = E_m$$

$$E = E_m e^{-\alpha t} \longrightarrow \qquad 10$$

Equation 10 gives the decay of sound energy density with time 't' even after the source is cut off. It is exponentially depressing function from maximum energy(Em) as shown. The growth and decay of sound energy together is represented in the figure.





3.4 PROOF OF RREVEREBARTION TIME(T)

According to Sabine, the reverberation time is defined as the time taken by a sound to fall to one millionth of its initial value, when the source of sound is cut off.

Time taken for E to be equal to $E = \frac{E_m}{10^6}$

Therefore condition is at t = T; $E = \frac{E_m}{10^6}$

Therefore equation 10 becomes $E = \frac{E_m}{10^6} = E_m e^{-\alpha t}$

$$10^{-6} = e^{-\alpha t}$$
$$10^{6} = e^{\alpha t}$$
$$\alpha t = \log_e 10_6$$
$$\alpha t = 6 \log_e 10$$
$$\alpha t = 6 \times 2.3026 \log_{10} 10$$
$$\alpha t = 6 \times 2.3026 \times 1 \longrightarrow \quad 11$$

We know $\alpha = \frac{vA}{4V} \longrightarrow \quad 12$

Substituting equation 12 in 11, we get

$$\frac{vA}{4V} T = 6 \times 2.3026$$
$$T = \frac{4V \times 6 \times 2.3026}{vA}$$

We know the velocity of sound in air at room temperature v = 330m/s

$$\therefore T = \frac{4 \times 6 \times 2.3026}{330} \times \frac{V}{A}$$

$$T = \frac{0.167V}{A}$$

$$T = \frac{0.167V}{\sum as} \longrightarrow \quad 13$$

Where $\sum as = a_1 s_1 + a_2 s_2 + \ldots\ldots\ldots .$

Equation 13 represents the Reverberation time, which depends on the three factors viz,

i. Volume of the hall(V)
ii. Surface area(S)
iii. Absorption coefficient(a) of the materials kept inside the hall.

Among these three actors volume is fixed. Therefore, the reverberation time can be optimized by either varying the surface area of the reflecting surfaces or the absorption coefficient of the materials used inside the hall.